

# Dunaakvics

A Dunaújvárosi Főiskola online folyóirata 2013. I. évfolyam V. szám

Műszaki-, Informatikai és Társadalomtudományok

I. ANDRÁS – M. RAJCSÁNYI-  
MOLNÁR – G. FÜREDI  
Firms with Corporate Res-  
ponsibility: The Practical  
Levels of Interpretation  
in the Intersection of CSR  
and Cafeteria



MARTIN NÉMETH  
Sketching rational  
functions



B. FEKETE – A. SZEKERES  
Finite Element Modeling  
of the Second-sound  
phenommenon



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## **Martin Németh: Sketching rational functions**

### **Absztrakt:**

Jelen dolgozatban azt vizsgáljuk, hogyan alkalmazható a MATLAB program függvények ábrázolására. Bemutatunk egy MATLAB szkriptet (ratfun.m) és áttekintjük a Dunaújvárosi Főiskolán a matematika tárgyak keretében megismert módszereket. Számos feladatra egyéni megoldást mutatunk.

**Kulcsszavak:** MATLAB, függvények vázolása, szkript

### **Abstract:**

In this paper, we examine how to use MATLAB in order to graph functions. We introduce a MATLAB script (ratfun.m) and we review the mathematical methods taken from the courses of the College of Dunaújváros. Alternative solutions will be provided for several problems.

**Keywords:** MATLAB, sketching functions, script

## Balázs Fekete – András Szekeres: Finite Element Modeling of the Second-sound Phenomenon

### Absztrakt:

A mérnöki gyakorlatban a hővezetés matematikai leírására a Fourier-féle egyenletet használjuk, mely azonban elvileg hibás, ugyanis az következik belőle, hogy a hő végtelen sebességgel terjedhet, mely a differenciálegyenlet parabolikus jellegéből adódik. Az elmúlt évtizedekben ennek az ellentmondásnak a kiküszöbölésére számos általánosított hővezetési egyenletet publikáltak. Ezek a klasszikus hővezetési egyenlet módosított változatai, melyek a módosító tagok következtében hiperbolikus típusúak. A módosított egyenletekkel leírt hőtranszport jelenség hullámterjedés jellegű, szemben a Fourier (és a vele analóg Fick egyenlet) diffúziós jellegével. A hiperbolikus hővezetést a szakirodalom Second Sound jelenségnek nevezi. Előadásunkban összefoglaljuk a klasszikus hővezetési egyenlet módosított változatait és bemutatjuk a Cattaneo-Vernotte féle egyenlet végeeselemes megoldását COMSOL szoftver segítségével 3D modellre.

**Kulcsszavak:** hővezetés, módosított hővezetési törvény, relaxációs idő

### Abstract:

The heat propagation task described by the Fourier's law leads to the parabolic differential equation and as a consequence to unlimited speed of propagation. During the last three decades, nonclassical theories free from this drawback have been formulated. These new theories apply modified versions of the classical heat transport equation and involve hyperbolic-type heat transport admitting finite speeds of thermal signals. According to these theories, heat propagation is to be viewed as a wave phenomenon, instead of diffusion phenomenon. A wavelike thermal propagation is referred to as second sound. We summarize the nonclassical theories and implement the Cattaneo-Vernotte type of equation in COMSOL Multiphysics and solve the problem with 3D model.

**Keywords:** heat conduction, modified heat conduction, thermal relaxation time



ISTVÁN ANDRÁS – MÓNICA RAJCSÁNYI-MOLNÁR – GÁBOR FÜREDI

## *Firms with Corporate Responsibility:* The Practical Levels of Interpretation in the Intersection of CSR and Cafeteria

### Introduction

For employees with a limited competence in CSR and human resource management, certain fields often overlap. The boundaries between the areas of wage types, cafeteria type fringe benefits and CSR-related areas sometimes become difficult to delineate. In our paper, we propose the concept of a firm with corporate responsibility and its interpretation. We will also seek answers to paradigmatic questions of a partly philosophic, partly human resource nature.

We suppose that for a prospective employee, there are no sharp differences between the concepts of incentive management and CSR (on the relation of CSR and HR c.f. Cohen, 2010; Fertetics, 2009; Filius, 2010; Sharma et al., 2009; Strandberg, 2009).<sup>1</sup> We set up a focus group interview. Its participants were graduating students who took the course Social Dialogue and Corporate Social Responsibility; they participated for at least ten weeks, their conceptual competence was developed, and they were interested in the topic.

In the first section of the paper, we create a framework constituted by the challenges of global human resource management; afterwards, we adjust paradigms of CSR and internal CSR. After this, we map the relation of

<sup>1</sup> Note: literature does not especially feature incentive management within the context of human resource management and CSR, either; it does discuss the field of management incentives (c.f. Ben-Amar-Smaili, 2012; Callan-Thomas, 2011; Kumar, 2011; Mahoney-Thorne, 2005; McGuire et al., 2003).

CSR to HR, the history of the changing progress of cafeteria type benefits within the framework of incentive management. We interpret firms' community-generating internal CSR activities, and discuss the survey we did among students. We consider the implications of the results of the focus group interviews in the light of philanthropic responsibility.

## **Challenges of global human resource management in quality assurance**

The theory of sustainable development is highly questionable. The Earth is too small, resources are finite and they are partly irreparable. Humanity is self-destructive. Every enterprise that takes soil, air, human resources etc. from its environment will naturally have to give something back. Global challenges centre on profit maximization; enterprises visualize progress in expansion.<sup>2</sup> An undeniably characteristic feature of our age is the scenario where a global (transnational, multinational etc.) enterprise is confronted with a local community that takes issue with the operation and activities that pollute the environment—even prior to the firm's entering the site. The "sharpness" of the questions is proportionate with the economic and socio-cultural development of the targeted area where the investment is bound to happen: several economically underdeveloped areas are happy to host even such industries that pollute the environment. Unbelievable destruction has been done in such defenseless areas as, for example, at the delta of the River Congo, which has turned into a lifeless swamp suffocating in petrol pollution.<sup>3</sup>

At the same time, we may credit global processes with the fact that in business partnerships, Quality Assurance has become a basic condition for a company to be "presentable". It is common knowledge among actors of economic life that Quality Assurance frameworks reflect the state of development in an enterprise. Frameworks of Quality Assurance, however, may have different levels (Figure 1).

On the first level, the enterprise regulates the process of the production of goods, avoiding the production of excessive wastrel; they set up the conditions of producing products in good quality (ISO1). On the second level, Quality Assurance expands to the monitoring of customer satisfaction, which is also regulated systematically. Learning about customers' opinion is indispensable both in the field of product development and in sales (ISO2). The supplier inevitably enters the next level, since

customers' demand can be only satisfied through suppliers (ISO3). Merging these three levels into a single process, however, is still only about the product. At a more advanced level of awareness, we expand the framework of Quality Assurance to our regulated environmental relationships, and set up basically environmentally friendly frameworks. At this level, the motive (ISO4) appears, and conditions and assists the embedding of the company. CSR refers to a framework that goes beyond environmental protection and regulations; it realizes the firm's voluntary social responsibility (ISO5)<sup>4</sup> with or without the application of GRI<sup>5</sup>. CSR – in the spirit of GRI markers – is, on the one hand, a more extensive area than environmental management; on the other hand, it provides internal intersections reflecting corporate processes. From this perspective, CSR is a “quality assurance system” that goes beyond ISO4 categories, and is characterized by areas also applied in GRI. In its CSR, firm presents its voluntary social responsibility towards both its employees and society.

It will take some time to reach the point where CSR activities will be the condition of “being presentable”, as are lower level operations of quality assurance.<sup>6</sup> The shortage of resources, economic tensions and civil consciousness will presumably force the firms towards an effectively responsible disposition.

## **The paradigms of CSR and internal CSR**

There are various approaches to the voluntary social responsibility of corporations.<sup>7</sup> We also consider CSR the activities of internal voluntary responsibility focusing on the company's employees themselves. The reason for the existence of internal CSR is also visible in GRI. Its economic aspect includes the wages and allowances of employees, while its social dimension includes fair employment, the realization of human rights, and product responsibility. Each field mentioned above constitutes a subset of human resource management, or may be related to one. Accordingly, only those firms may be called socially responsible that can gain accreditation at the various levels of employment through the GRI index. Economic, environmental and social aspects cannot do without a systematic approach to the firm's internal human relations framework. Yet we do not interpret internal CSR through the

summary of HR data received through the GRI index; we use a significantly different methodology.

## **The concept of the corporately responsible firm**

According to the argument above, there is internal CSR,<sup>8</sup> it is not, however, the statistical message of the internal corporate relationships of the GRI indexes. A firm is corporately responsible if its activities target at the universal well-being and satisfaction of its employees. The main features of this activity are voluntariness, separation from the framework of wages and cafeteria, and responsibility. We consider a corporate nursery set up for the children of the firm's employees a token of internal CSR, but we do not consider a wellness holiday season ticket provided as part of cafeteria benefits to be CSR. Later we will define in detail the indicators of establishing boundaries. The key concept of internal CSR – just like for external CSR – is voluntariness. The company provides something for its employees, and does not expect anything in return, thus contributing to their well-being and satisfaction.

Within CSR, voluntariness is often received with suspicion; it is often pointed out that a company does everything for profit.<sup>9</sup> After all, by contributing to the well-being of an employee, a company can reasonably expect a higher-level performance. If employees can leave their child in the corporate nursery, they will not have to bother about application and enrolment to a nursery school, and they don't have to travel extra because of this. They can work for the firm balanced and calm; it saves time and energy for the company. Suspicion, however, is mostly groundless: a firm has every right to expect its employees to arrive on time and work with total dedication even if it does not set up a nursery school under its roof. In a wider context, a firm is capable of giving back something from its profit for internal or external CSR activities, if it is able to produce the profit rate that is needed for this. In this sense, internal CSR is a kind of special "profit-sharing" established primarily for the employees and, indirectly, it has an impact on the environment of the firm.

## The relationship between CSR and HR

The socially responsible operation of companies – as we have already pointed this out above – is still in its infancy. In a significant number of CSR-committed organizations, activities have been limited to various types of support and corporate donations.<sup>10</sup> The aim is to generate a corporate culture that is based on CSR principles and values, and whose transformation is shaped by employees and, consequently, by HR functions as well.<sup>11</sup> HR has to step up as protagonist in the support of CSR activities, and in the shaping of a proper politics of conduct (Sharma et al., 2009). “CSR=HR=PR”. If employers are not committed, CSR turns into a PR tool. The integrity of the company is destroyed if it turns out that it is “preaching water, drinking wine” (Mees-Bonham, 2009).

According to Mandy Fertetics (2009), HR is not exclusively influenced by the values and principles of CSR; the environment, corporate strategies and competitiveness also have an impact on it – mutually, as a back and forth effect (c.f. Figure 2).

According to Mandy Fertetics (2009), CSR is manifest on three levels with relation to human resource management (c.f. Figure 3):

- within HR operations,
- with the support of HR, in the realization of the responsible operation of the other areas of operation
- in the realization of HR-based voluntary CSR actions.

Though the three levels do not necessarily have to follow one another, it is recommended to build bottom-up in order to increase the consciousness of CSR processes, and to secure credibility.

## On cafeteria type benefits

The meaning<sup>14</sup> and nature of cafeteria type fringe benefits have undergone significant changes since its introduction in Hungary. In the 1990s, cafeteria in fact referred to allowances beyond wages; by now it has become an organic part of the wage. The turbulent cafeteria movements of the 1990s were

articulated within the framework of human resource management interests as incitement management (c.f. Farkas et al., 2003; Karoliny-Poór, 2010; Poór, 2011). Managers' bonus-cafeteria packages were issued, beyond wages, as a form of reward, as a second degree reward beyond the actual one that ranged from a company car, to limited mobile phone use, exclusive health and wellness treatments, and studies abroad. On the non-management level, companies "swept" together all the expenses they spent on their employees, and created cafeteria wage volume and cafeteria schemes in a proportionate manner.

After its euphoric headway in the nineties, cafeteria was losing its magic in the early 2000's, and became an extra wage allowance<sup>15</sup>. Employees, the employer and interest groups re-conceptualized the frameworks of cafeteria: low-cost elements came into focus. We know of wage negotiations where wage development and cafeteria improvement were treated together, yet they arrived at significantly different rates (e.g. 8% increase in basic wages and 12% increase in the cafeteria scheme in certain activity scopes or on average).

By now, cafeteria has become an "intra wage" item; an allowance package that constitutes an integral part of the employee's income, where the real issue is often the matching of individual demands and corporate interests, and the calculation of the consequences of abiding by tax regulations. Cafeteria today is a segment of income that constitutes part of the wage agreement: and is not an optional extra element, but rather a compulsory "must". It is quite frequent that basic wages are kept at an extremely low level through large doses of cafeteria (for example minimum wage complemented by a considerable holiday voucher) in order to optimize the employer's employee-related fee payment obligations.

With reference to cafeteria, the key question is the employees' freedom of choice. The degree of freedom is well marked by the orientation of choice, by the width and depth of the "menu" offered.<sup>16</sup>

The role of the state is marked both in the choices applied to cafeteria, and in channeling individual choices. The various state preferences are manifest through tax regulations and various legal rules. Cafeteria can be interpreted as a guide to the state's disposition, since the nature of different tax allowances, preferential bonuses and severely taxed wage elements all serve as a compass both for employers and employees.

The “reward nature” of cafeteria has been fading away partly because of the governmental preferences mentioned above, and partly because of its increasing intra wage role, also mentioned before. Markers of “intra sponsorship” (gift ticket to the theatre for employees etc.), however, do have an incentive impact.

Our earlier surveys on incentive management show that some employers incorporate holiday and sick leaves into cafeteria. However, a firm that merges employers’ cafeteria allowances with their sick leave gives the impression of a socially irresponsible company. First, consistent employee presence is rewarded through cafeteria; second, sick leaves are punished through the cutting of cafeteria. Since the cutting of allowances may prompt employees to work even when they are sick, this method may be considered unethical.

### **Possible interpretations of the intersections of cafeteria and internal CSR**

Among communication and media studies BA majors at the College of Dunaújváros, the most popular course among students is business communication. We pay special attention to the involvement of students in surveys. During one of our regional surveys (Complex Cultural Discourses Survey (4K)), we invited about fifty students to join the research. Tasks included conducting interviews at the mall, content analysis, and conducting interviews with senior management at firms. This enabled our students to have an insight into enterprises, which later served as points of discussion at the seminars.<sup>17</sup>

In response to student feedback and experiences, we created two focus group discussions on week 11 of the term (2011). On each occasion, the authors, who were also the professors of the course, and seven students participated in the discussions. More students volunteered but, eventually, they got to select who would participate.

Some students already had been employed before or during the investigation. Their cafeteria and CSR awareness had developed either because of their own experiences, or that of their families. Some students were not merely “tiptoeing” on the fine lines of CSR and cafeteria, but also knew its “delicacies.” On the whole, focus group interviews were based on a sound conceptual framework. We illustrate

ted the discussions through examples.

Features of boundaries (interfaces) were clearly marked in both cases. Transition was considered exciting from one aspect: when can cafeteria become internal CSR? We have summarized the theoretical conclusions of the focus group discussions below:

- Voluntariness, voluntarily “provision,” is a key question both in case of cafeteria and in case of internal CSR. Whenever students sensed expectations, they became dismissive. They had a crushing opinion on connecting sick leave to cafeteria. They were also outraged by the idea of treating CSR as internal PR.
- The idea of higher principles came into focus. The protection of life, job security, health, children and community became key words around which a unique framework of values emerged. They were dismissive about examples that prioritized the compulsory provision of tools for reasons of job security. Ergonomics and personal employee comfort (workplace well-being) were accepted as the only valid considerations. They accepted the demand for clear water supply, for schooling benefit (to be received by both employee and their children). They considered the role of profit a possibility that might open the path towards the achievement of higher goals (cancer check, recreation etc.). They found the corporate nursery one of the nicest solutions – a combined form of cafeteria and CSR.
- They did not think much of solutions in the service of the daily routine (meals, season ticket for the bus). Students’ expectation marked a future-orientedness (for example talent enhancement schemes). Life-support CSR events also feature future-orientedness; students were not bothered by the fact that meals on Family Accident Prevention Day were financed through cafeteria, and they would be detracted from the employee’s cafeteria budget.
- An expected exclusiveness is also characteristic (the luncheon voucher that can be spent at the canteen should not be CSR by any means; when, however, the possibility of bio-food emerges, it immediately acquires an aura of CSR, since bio-food promotes the health of employees in the long run, which also indicates future-orientedness).
- Students have attributed special significance to internal CSR and to the real value of cafeteria.
- They oppose that junk objects and cheap programs should feature at CSR events for children. To

get a package from Santa Claus is not an issue any more, what matters is the content of the package: is it real chocolate or some cocoa dollop, is the wrapping plastic, or recyclable? They dismiss the idea of inviting celebrities even if they would make a particular event more attractive. Their sense of authenticity and truth proved infallible in case of charitable events.

- They consider loyalty an exchange value equivalent both in case of cafeteria and in case of CSR. An employee's loyalty increases on account of employee-satisfaction, he or she will perform more efficiently, will be more generous with his or her time for the firm.

Cafeteria and internal CSR have interfaces. If an employer provides something in form of cafeteria that conforms to the principles above, we may rightly consider it a corporately responsible disposition. There may be noble goals among the priorities of human resource management that point at the direction of voluntary social activities, and can be integrated within the framework of cafeteria. The CSR interface<sup>18</sup> is also meaningful when we talk about allowances outside cafeteria.

Cafeteria provides the employee with choices; whether the employee makes use of the prospective CSR-type choices does not primarily depend on the employer. The employer, naturally, may rely on the means of orientation, but it is not wise in most of the cases "to tell people what is good for them."

### **Conclusion: philanthropy is not a question of money**

Philanthropic responsibility is on the top level of CSR responsibilities. We have met a CEO who started a campaign in which the whole Ltd. and half the town were collecting plastic bottle caps because the child of a doorman at the firm was seriously ill, and therapy was only available abroad. Is this CSR? Is it internal CSR? Is it external CSR? Is it a campaign of individual charity? These are questions worth asking only when we train students to be receptive to complexities.

Material goods and comfort gain value in a global world. Yet we should not forget about the fact that human factors also feature in the pyramid of needs. Employees and society have a basic demand to communicate with players of economy. The demand for a responsible attitude is becoming increa-

singly appreciated both on the level of the individual and the local environment. The measure of life quality is less and less material, and increasingly ethical and cultural. Employees' demand for a CSR-based self-management has also emerged, as several authors point it out (see Kun, 2009).

The IQ-EQ-SQ19 curve may serve as a guide in human resource management for leaders with regards to philanthropic responsibility. A good manager is accepted when he or she is socially responsible and open. And so is the company.

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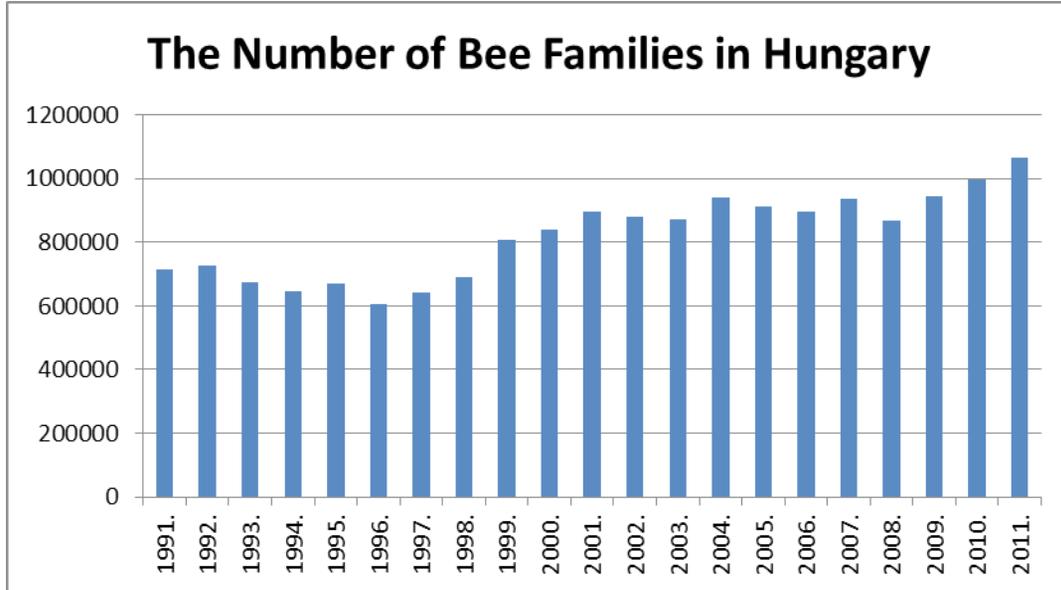
## *Sketching rational functions*

### Introduction

Between two - or more - quantities the relationship can be frequently determined. For instance, the number of the insect called *Apis mellifera L's* (*honey bee*) is changing regularly in Hungary. Certainly, the number of the bee families is a defined (not negative) number at any given time, that is to say, there exists a function that relates the number of bees to the time. (It would be a challenge to describe it with formula). The following chart shows the number of bee families and their fluctuation in Hungary (Tóth, 2012).

Year	The Number of Bee families in Hungary
1991.	716394
1992.	725615
...	...
2009.	943824
2010.	997022
2011.	1065860

The histogram made from the data of the chart shows the changes more visibly.



The relation between the variables can be shown in a chart as well as formulas. In either case it is practical and useful to represent the coherent pairs on graph for better understanding, because people often interpret visual information much faster than a formula. Who does not remember from their student years the line: „As the graph shows, it is obvious that...”

In applied research, there are a lot of examples for the usefulness of knowing some properties of a function. For example, in the theory of differential equations, the number of solutions of a given system can be determined by a simple procedure using the geometric properties of particular graphs (Simon P. et al, 1999); (Nagy B, 2009).

Another interesting field of research is geometry. In this case, using the behaviour of a particular function leads us to the creation of a body with odd Hadwiger-number (Joós, 2008).

The students of the College of Dunaújváros study mathematics for several semesters, including the

analysis of functions. For this analysis they need to be able to graph related functions (the derivative of the given function). In the course Mathematics I., there are "representation of functions", "sketching rational functions", "interpretation of multivariable functions". The representation of functions and graphs will be a necessary tool in the course Mathematics II. for the description of statistical data, as well. In the course Mathematics III., it appears in the solution of nonlinear equations and differential equations.

At the Collage of Dunújváros the computer program MATLAB (MATLAB 2013) is used in mathematics courses. MATLAB (MATrix LABoratory) is a numerical computing environment and fourth-generation programming language developed by MathWorks. This program allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interacting with programs written other languages.

Although MATLAB is intended primarily for numerical computing, an optional toolbox (using the MuPAD symbolic engine) allows access to symbolic computation. In 2013, MATLAB has more than a million users across industry and academia.

There are many ways to display data graphically by using MATLAB. We can choose from 70 different methods (49 Standard Plot; 12 Customizing Plots; 9 Advanced Plots) to create our own MATLAB project.

## Sketching rational functions

At the College of Dunaújváros the course Mathematics I. contains "sketching rational functions", which uses the pen-and-paper method with the aid of a computer (Writing m-files or MATLAB scripts however are not mandatory requirements). We will see that, mixed with this pen-and-paper method, the plot command of MATLAB will be more useful and effective than the pen-and-paper method or the plot command alone.

The algorithm of pen-and-paper method, which is the basis of the script ratfun.m, is shown in the

following example.

Sketch the graph of a given rational function.

Solution: We suggest using the algorithm below.

1. Determining the domain of the definition of function  $f$ .
2. Finding the zeros of function  $f$ .
3. The approximation of function  $f$  by the zeros.
4. Finding limits at infinity and minus infinity.
5. Finding limits at the points of discontinuity.
6. Sketching the graph.

This method is complicated and tedious, it is difficult to calculate every detail. It is however straightforward. A computer with MATLAB is the appropriate tool to solve a problem like this. It's important to realize that the plot command in and of itself is not sufficient (although some other commands like axis may help create more informative figures). To understand why the previous statement is true, let's consider the following problem:

Sketch the graph of the given functions, and compare the graphs.

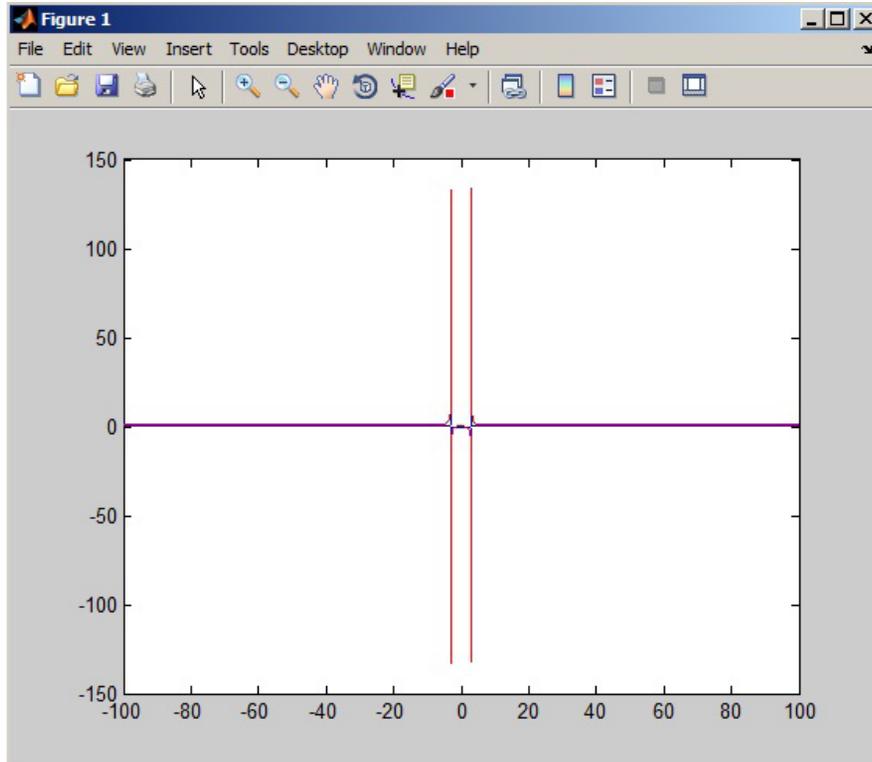
$$f : \mathbb{R} \rightarrow \mathbb{R}, x \mapsto \frac{1}{x^2 - 9}$$

$$g : \mathbb{R} \rightarrow \mathbb{R}, x \mapsto \frac{x^2 - 1}{x^2 - 9}$$

Without investigating these functions, it is hard to know on which interval we should sketch them. It may be a good idea to try a large interval, for example  $[-100; 100]$ . Since we would like to have a detailed graph it seems reasonable to have a small stepsize, say 0.01. (Later on we will see, why the large interval and the small stepsize contradict each other.)

To plot the graph of the functions, we apply `>> x=-100:0.01:100; >> plot(x,1./(x.^2-9))`  
`>> hold on >> plot(x,(x.^2-1)./(x.^2-9))`. There are significant differences between the two graphs, but it is challenging to find them.

As it can be seen from the following figure, graphing the two functions in the same coordinate system doesn't solve our problem (even if function f is in red, and function g is in blue...).



Based on the arguments and the figure above, we can propose that large interval and small stepsize together can't result in informative figures. The graph of the function  $f(x)=1/(x^2-9)$  is hard to sketch

because of the difference in the order of the magnitude of values (e.g.  $x=-100.0000$  means  $f(x)=0.0001$ , and at  $x=2.9990$   $f(x)=-166.6944$ ).

Of course, in order to have informative graphs, there is an appropriate interval and stepsize for every function. But first we have to carry out a similar calculation as we saw before.

A simple solution is to solve the problem in general, that is to write a MATLAB script, which calculates the necessary characteristics.

In this work we only consider the case when the denominator and the numerator of the function is quadratic. The problem can be solved in a similar way for arbitrary rational functions. Using simple MATLAB commands, a script (`ratfun.m`) that plots rational functions and calculates the corresponding characteristics of a function (exported into `funchar.txt`) can be written.

## Testing the script

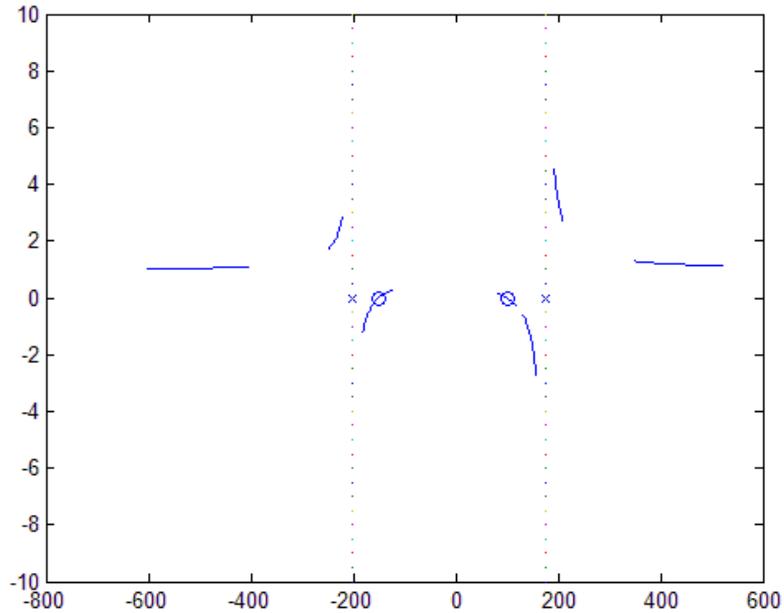
In what follows, we introduce the results achieved by `ratfun.m`, and compare them to the graphs created by the simple `plot` command.

Test 1:

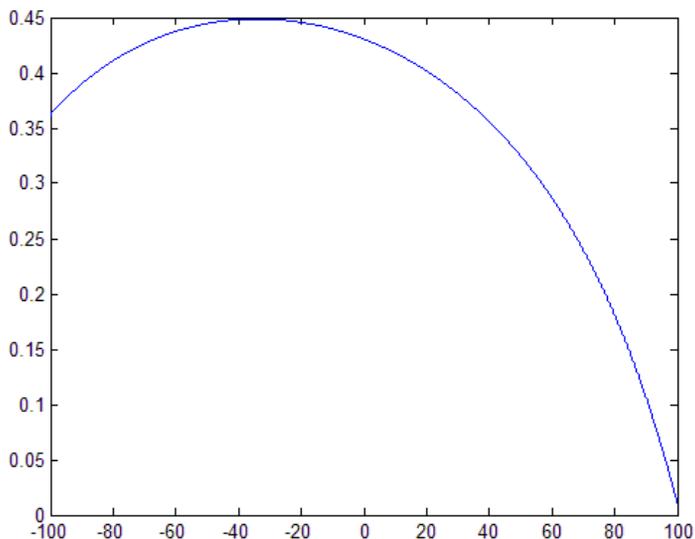
In this example the numerator and the denominator of the rational function is quadratic. Both have two distinct real roots, and the fraction can not be simplified. The text file `funchar.txt` includes the following results. (In what follows "Inf" means infinity.)

We investigate the function below:  $(x^2 + 49x - 15150)/(x^2 + 26x - 35175)$ . The points of discontinuity are -201 and 175. The zeros of the function are -150 and 101. The limit at infinity is 1.

The limit at minus infinity is 1. At the point of discontinuity -201 the limit of the function from the right is  $-\text{Inf}$ . The limit of the function in this point of discontinuity from the right is  $\text{Inf}$ . At the point of discontinuity 175 the limit of the function from the right is  $\text{Inf}$ . The limit of the function from the right at this point is  $-\text{Inf}$ . The sketch of the function can be seen on the next figure using `ratfun.m`.



Without any previous calculations, the user has to pick an interval randomly in order to plot the function. If this random interval is e.g.  $[-100, 100]$ , neither the zeros of the function, nor the points of discontinuity and the limits are efficient, as can be seen from the next figure.

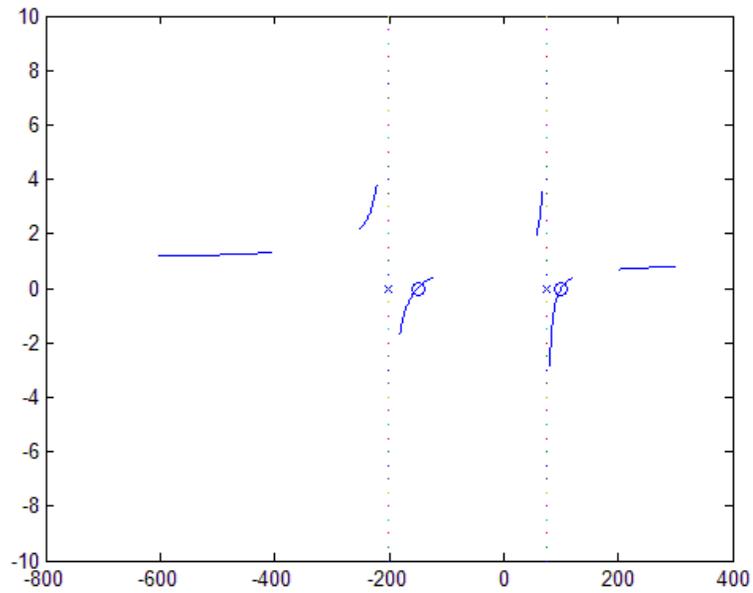


## Test 2.

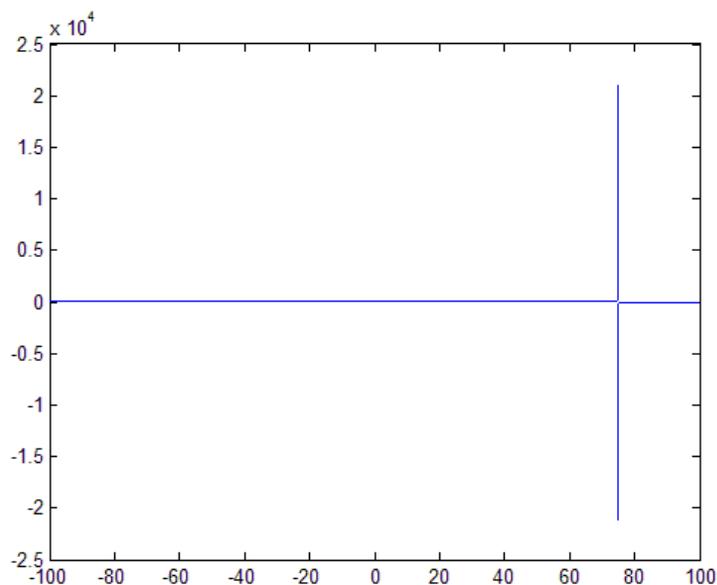
In this example the numerator and the denominator of the rational function is quadratic, both have two distinct real roots, and the fraction can not be simplified. The text file funchar.txt includes the following results.

We examine the following function:  $(x^2 + 49x - 15150)/(x^2 + 126x - 15075)$

The cut-off places are -201.0000 and 75.0000. The zeros are -150 and 101. The limits at infinity is 1. The limit at minus infinity is 1. At the cut-off place -201.0000 the limit of the function from the right is  $-\text{Inf}$ . The limit of the function at the cut-off place from the left is  $\text{Inf}$ . At the cut-off place 75.0000 the limit of the function is  $-\text{Inf}$  from the right. The limit of the function at the cut-off place from the left is  $\text{Inf}$ . The sketch of the function can be seen on the next figure using ratfun.m.



Using MATLAB command plot (on the random interval  $[-100, 100]$ ) nor the zeros of the function, nor the points of discontinuity, nor the limits are efficient.



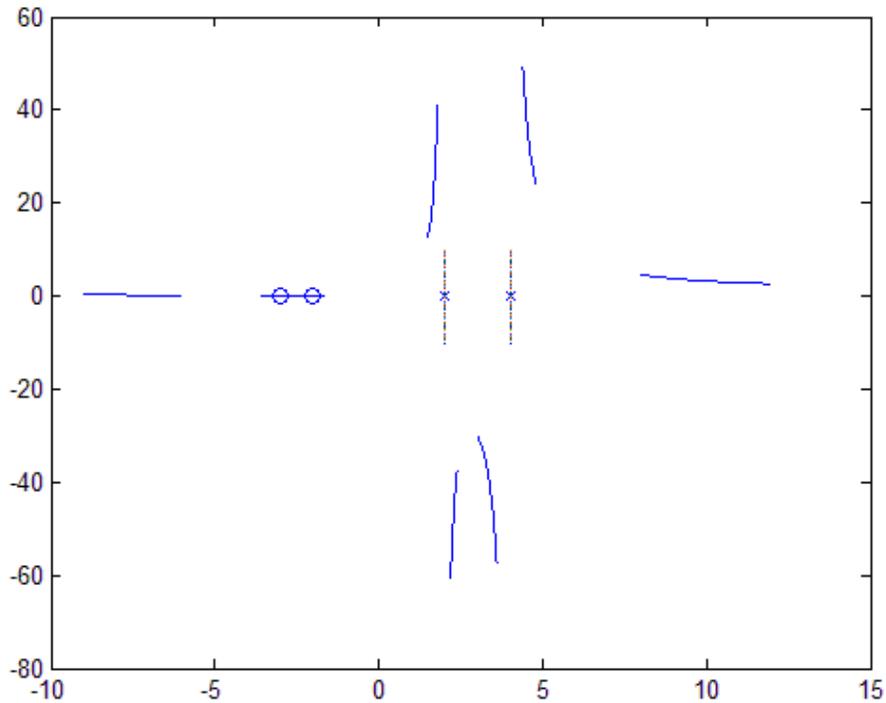
### Test 3:

In this example the numerator and the denominator of the rational function is quadratic, both have two distinct real roots, and the fraction can not be simplified. The text file funchar.txt includes the following results.

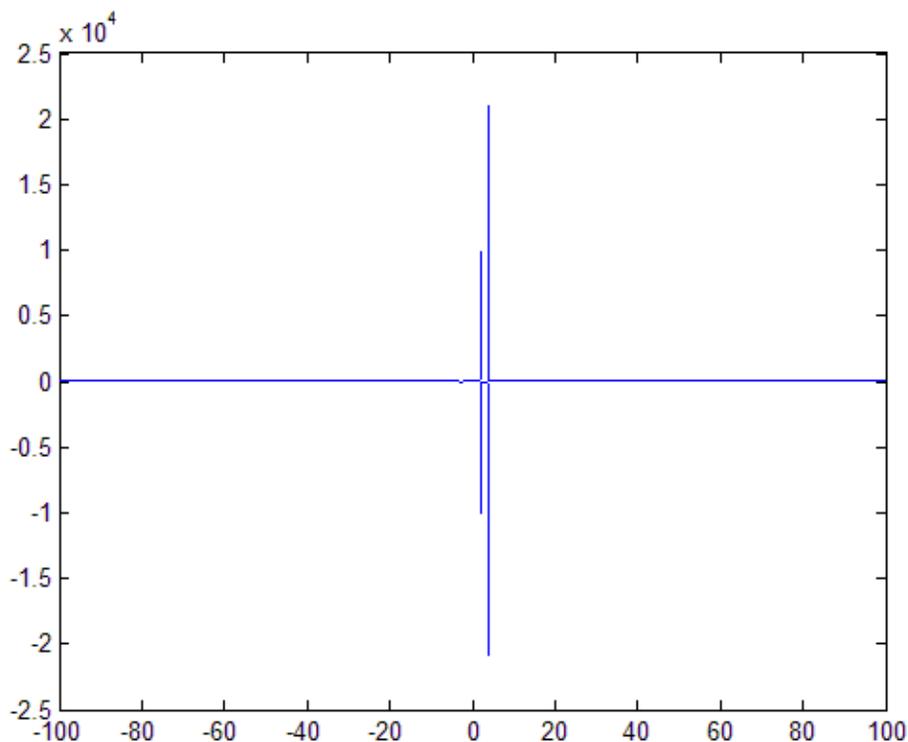
We investigate the function below  $(x^2 + 5x + 6)/(x^2 - 6x + 8)$ . The function is not defined if  $x=4$  or  $x=2$ . The zeros of the function are  $-3.0000$  and  $-2.0000$ . The limits at infinity and minus infinity are 1. The points of discontinuity are 2 and 4. The limit of the function at  $x=4$  from the right is  $\text{Inf}$ . The limit of the function in this point from the left is  $-\text{Inf}$ . At the point of discontinuity 2, the limit of the function from the right is  $-\text{Inf}$  and the limit of the function in this point from the left  $\text{Inf}$ .

Using the results of the calculation, ratfun.m defines the domain of definition for function f and plots the function on this interval. In the window Figure 1 the following function can be seen.

The ratfun.m MATLAB script results



Using the chosen interval  $[-100, 100]$  MATLAB command plot nor the zeros of the function, nor the points of discontinuity, nor the limits are efficient.

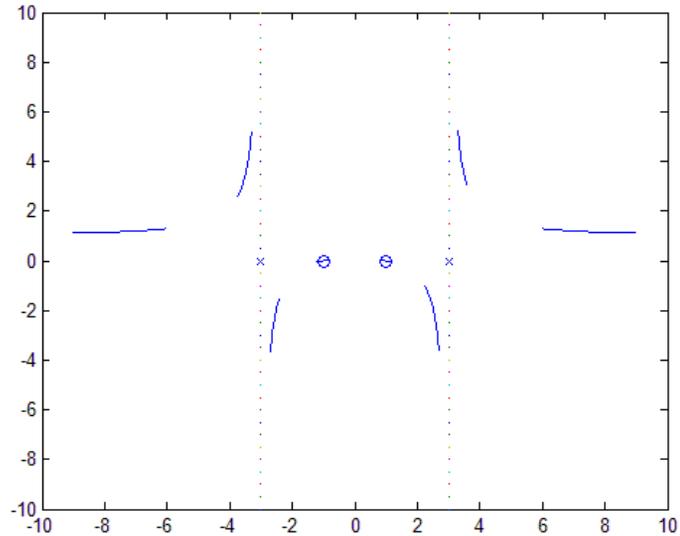


#### 4. test

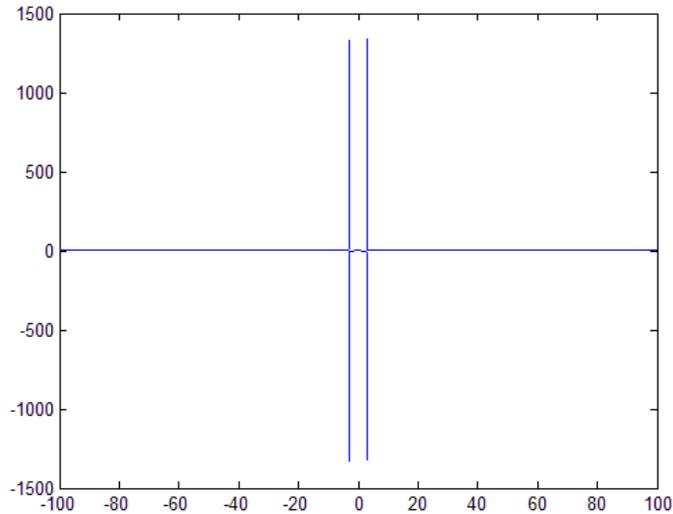
In this example the nominator is constant and the denominator of the rational function is quadratic, and has two distinct real roots. The text file `fvjellemzoi.txt` includes the following results.

We investigate the function below  $(x^2 - 1)/(x^2 - 9)$ . The function is not defined in 3 and -3. The zeros of the function are -1 and 1. The limits at infinity and minus infinity are 1. At the point of discontinuity 3 the limit of the function from the right is  $\text{Inf}$ . The limit of the function in this point from the

left  $-\infty$ . If the point of discontinuity is  $-3$ , the limit of the function from the right is  $-\infty$ . The limit of the function in this point from the left is  $\infty$ . The `ratfun.m` MATLAB script results



Using MATLAB command `plot` nor the zeros of the function, nor the points of discontinuity, nor the limits are efficient.



## Detected deficiencies, and opportunities for further development

In the following, we present the deficiencies that we found during testing.

The `ratfun.m` MATLAB script doesn't give more useful results than the `plot` command if the points of discontinuity "accumulate" next to the zeros of the function. This problem can be solved as a programming exercise. We will discuss this in another paper.

Also, it would be practical to access the input as a parameter. This problem can be solved as a programming exercise as well by using the possibilities of MATLAB. MATLAB scripts that use incoming parameters can be written.

The script `matfun.m` can be extended over any polynomial of any order. We don't discuss this in detail here, but all that needs to be adjusted is the input.

MATLAB by default works with complex numbers. For example if the denominator is  $x^2+1$ , it cal-

culates two complex points of discontinuity, which brings up further problems.

There are other development opportunities as well.

## Summary

In this work we introduce some results with a simple script (`ratfun.m`) written to sketch rational functions with MATLAB. The script unifies the advantages of the pen-and-paper method of sketching functions with the possibilities of MATLAB. It is found that `ratfun.m` results more effective graphs, than the `plot` command in itself.

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## *Finite element modeling of the second-sound phenomenon*

### **Introduction**

The scientist dealing with thermo-elasticity sooner or later is faced with questions involving the modification of the law of heat conduction. In the case of analyzing dynamical processes he either accepts one of the modified laws of heat conduction or he may try to solve this problem relying more or less on his own resources. One of the most simple and accepted approaches that surmounts the limitation of Fourier law was suggested by Cattaneo (1948) and independently by Vernotte (1958). It consists of modifying the heat flux equation, incorporating the finite propagation speed of heat. After the theoretical background the finite element implementation of the Cattaneo-Vernotte equation were carried out.

### **Theoretical background**

#### **Modified equations for heat conduction**

The classical theory of heat conduction is based on the equation:

$$\mathbf{q} = -k\nabla T, \quad (1)$$

for the heat flux  $\mathbf{q}$ , which combined with the conservation of energy equation:

$$\rho \frac{\partial e}{\partial t} + \nabla \mathbf{q} = 0, \quad (2)$$

and the equation of state:

$$e = cT, \quad (3)$$

we get the well-known equation for T:

$$k\nabla^2 T = \rho c \frac{\partial T}{\partial t}. \quad (4)$$

This equation is a parabolic-type heat partial differential equation that allows an infinite speed for thermal signals. During the last six decades nonclassical theories free from this paradox have been published. These new models are mostly modified versions of the classical Fourier's equation and consequently involve hyperbolic-type heat transport equations admitting finite speed for thermal signals. According to these theories heat transport is a wave phenomenon rather than a diffusion type. These new theories are referred as theories with finite wave speeds or theories with second sound (the first sound being the usual sound).

The first model to remove the above mentioned paradox was proposed by Carlo Cattaneo and Pierre Vernotte in 1958:

$$\tau \frac{\partial \mathbf{q}}{\partial t} + \mathbf{q} = -k\nabla T. \quad (5)$$

The modified Fourier equation coupled with the energy balance equation leads to a hyperbolic heat equation:

$$k\nabla^2 T = \rho c \left( \frac{\partial T}{\partial t} + \tau \frac{\partial^2 T}{\partial t^2} \right). \quad (6)$$

Now the new material parameter appearing in the heat equation which referred as thermal relaxation time.

Some of other modification of the Fourier equation are the following:

$$\tau \frac{\partial \mathbf{q}}{\partial t} + \mathbf{q} = -k \nabla T + a_1 \nabla^2 \mathbf{q} + a_2 \nabla^2 \mathbf{q}, \quad (7)$$

$$\tau \frac{\partial \mathbf{q}}{\partial t} + \mathbf{q} = -k \nabla T + l \nabla \frac{\partial T}{\partial t}, \quad (8)$$

$$\mathbf{q} = -k \nabla T + a_2 \nabla^2 \mathbf{q}. \quad (9)$$

Here (7) is the Guyer-Krumhansl equation, (8) is the Jeffreys type equation, (9) is the Green-Naghdy type equation of heat conduction,  $l$ ,  $a_1$ , and  $a_2$  are material parameters. In Ref. 13 (Ván, 2011) was derived these models with the help of irreversible thermodynamics and the relations of the different terms was showed with their dissipative and nondissipative nature.

An other possible solution is the modifying the equation of state (Szekeres, 2004):

$$e = cT + c_1 T_t, \quad (10)$$

where  $C_1 = \tau C$ . It results is the (5) as well.

Since the thermal relaxation time is found to be very small, the variety of the researchers' opinion has wide range. Many authors have argued that the last term in (6) may be ignored in many practical problems. But some experts have employed the modified equation of heat conduction in their calculations to study some practically relevant problems and have found that, in heat transfer problems involving high heat fluxes or short time intervals, the hyperbolic heat conduction model gives significantly different result than the parabolic equation. Based on these studies we can conclude that, the modifying the Fourier's law is not negligible particularly when the elapsed time during a transient is less than about  $10^{-5}$  sec or when the heat flux involved is greater than  $10^3 \text{ W/mm}^2$ . Such conditions are not impossible for example in the nuclear industry. In our previous work (Fekete et. al, 2013) we showed that by one dimensional numerical modeling of the Cattaneo-Vernotte equation, depending on the value of relaxation time the difference between the solution of Fourier modeling and solution obtained by the modified equation can be significant.

## Physical meaning and mathematical model of thermal relaxation time

The material parameter  $\tau$  appearing in (5)-(7) has a definite physical interpretation. It results from the phase lag between the heat flux vector and the temperature gradient in a high-rate response. In other words  $\tau$  represents the relaxation time or build-up period for the initiation of heat flow after a temperature gradient has been imposed at the boundary of the domain. It states that heat flow does not start instantaneously, but rather grows gradually with a relaxation time  $\tau$  after the application of the temperature gradient. Thus, there is a phase lag for the disappearance of the heat flow after removal of the temperature gradient. So the relaxation time is associated with the linkage time between phonons (phonon-phonon collision) necessary for initiation of the heat flow and is a measure of the thermal inertia of the medium.

The above mentioned delayed behavior can be expressed as:

$$\mathbf{q}(r, t + \tau) = -k\nabla T(r, t) \quad (11)$$

where  $\mathbf{r}$  is the position vector of the elementary volume under observation (Ordóñez et. al, 2009). In the absence of a delayed response,  $\tau=0$  (11) reduces to the classical Fourier equation. After application of the linear Taylor series expansion to  $\mathbf{q}$ , we obtain the Cattaneo-Vernotte equation (5).

By focusing our attention on the mathematical representation of thermal relaxation time after some derivation of the modified heat equation (5) we get the following form:

$$\nabla^2 T = \frac{1}{D_T} \frac{\partial T}{\partial t} + \frac{\tau}{D_T} \frac{\partial^2 T}{\partial t^2}, \quad (12)$$

where  $D_T = \frac{k}{\rho c}$  the thermal diffusivity.

Focusing on the terms containing  $\nabla^2 T$  and  $\frac{\partial^2 T}{\partial t^2}$  we obtain:

$$\frac{\partial^2 T}{\partial x \partial x} + \dots = \dots + \frac{1}{\tau} \frac{\partial^2 T}{\partial t^2} \quad (13)$$

It is obvious that  $\frac{D_T}{\tau} t^2$  must have a dimension of square of a length. It suggests that the ratio  $\sqrt{\frac{D_T}{\tau}}$  must be a velocity-like quantity. Thus

$$v_t = \sqrt{\frac{D_T}{\tau}}, \quad (14)$$

with being  $v_t$  the thermal wave speed in the medium.

Both  $D_T$  and  $\tau$  are intrinsic thermal properties of the medium. The resulting thermal wave speed  $v_t$ , therefore, is also an intrinsic thermal property. It characterizes the thermal wave propagation the same way as the diffusion behavior characterized by the diffusivity. As termed by Chester, the reciprocal of the relaxation time,  $f = \hat{\Phi}/\tau$  is the critical frequency assign the activation of thermal wave behavior. When the collision frequency among molecules exceeds such a threshold, the wave behavior in heat conduction dominates over diffusion (Tzou, 1993). According to some recent works correlating the thermal wave theory to the microscopic model, the thermal wave speed  $v_t$  is related to the coupling factor  $G$  of the electron-phonon collisions and the volumetric heat capacities of the electrons  $C_e$ , and the metal lattice  $C_m$  through the equation (Tzou, 1993):

$$v_t = \sqrt{\frac{kG}{C_e C_m}}. \quad (15)$$

The thermal relaxation time with the microscopic parameter is:

$$\tau = \frac{1}{G \left( \frac{1}{C_e} + \frac{1}{C_m} \right)}. \quad (16)$$

The coupling factor of electron-phonon interactions depends on the number density of electrons, speed of sound, thermal conductivity, and Boltzmann constant. It is clear that the determination of the relaxation time and the thermal wave speed in microscopic point of view is impossible without some knowledge in solid state physics.

## Experimental methods on Non-Fourier heat conduction

In developing a suitable model for describing certain phenomena in engineering, establishment of a rigorous physical basis and comparison with experimental observations are equally important. Experimentally, the second sound was first detected by Peshkov in liquid helium in 1944. He was found the thermal wave speed to be equal to  $19 \text{ m/s}$ . Later, second sound was also detected in solid helium at temperatures about 0,6 K. Subsequently the phenomenon was also detected in NaF, NaI, and some other crystals. A typical value of the thermal relaxation time for metals at ambient temperature has been reported to be of the order of  $10^{-11} \text{ sec}$  (Tzou, 1993). In 1993 was reported by Majumdar that diamond has a high relaxation time ( $\hat{\omega}=10^{-3} \text{ sec}$ ) at 77K and exhibits hyperbolic heat conduction in a macroscopic sample. Due to the absence of a table for the thermal wave speed, table for the relaxation time for engineering materials is still absent at this point. The reason is that it is difficult to measure the thermal wave speed because of the very short time intervals. We have to sampling the quantities in a very high frequency range and ensure some special conditions e.g.: high temperature gradient or very low temperature.

Due to the above mentioned reasons very few experimental method was published on the validation of the modified law of heat conduction or experimental determination of the relaxation time. In Ref. 5. (Guillemet et. al, 1997) the theory and implementation of an experiment aimed at studying heat behavior on a short time scale was explained. By this measurement based on the forced Rayleigh scattering method, the Fourier law was well confirmed. The second sound phenomenon was investigated by Szekeres et. al. (1980) with thermal shock of long bar. They obtained that the thermal relaxation time lies at the order of magnitude of  $10^{-1} \text{ sec}$ , which is it considerably differs from the value given in literature.

## Implementation the Cattaneo-Vernotte equation in COMSOL Multiphysics®

The resulting non-linear governing differential equations (4)-(9) are not amenable to a closed form solution, and hence, a solution technique based on the finite element method is presented and implemented in COMSOL software. We have carried out an experimental setup to investigate the heat conduction phenomenon in 08H18N10T (AISI 321) austenitic stainless steel. This structural material is widely used in nuclear power industry and often affected by high rate thermal transients. The investigation were conducted on GLEEBLE 3800 thermomechanical simulator, the experimental setup is shown in Fig. 1. A special shaped specimen were designed as shown in Fig. 2.



Fig. 1

Experimental setup

*Tab. 1 Description of the experimental layout*

Nr.	Description
1	GLEEBLE 3800 simulator (max heating-cooling rate: $10^4 \text{ } ^\circ\text{C} / \text{sec}$ )
2	Test specimen
3	NI PXI computer (PC-based platform for high-rate sampling frequency temperature measurement)
4	Control computer with NI Labview® software

*Fig. 2**Geometry of the specimen*

The specimen were connected by the cylindrical surfaces to the jaws of the GLEEBLE simulator, which transfer the electrical current into the test sample. The resulting electromagnetic loss heating up the axial part of the specimen, nevertheless the heat get into the perpendicular part only by heat conduction. That is, this part behave as a long rod affected by thermal shock in the end.

The objective of the current section is to develop a finite element model of the experiment detailed above. The model were built up with triangular element applying refinement in the perpendicular part as shown in Fig. 3. A total of elements 29874 elements and 98954 nodes were used in discretizing the

mesh.

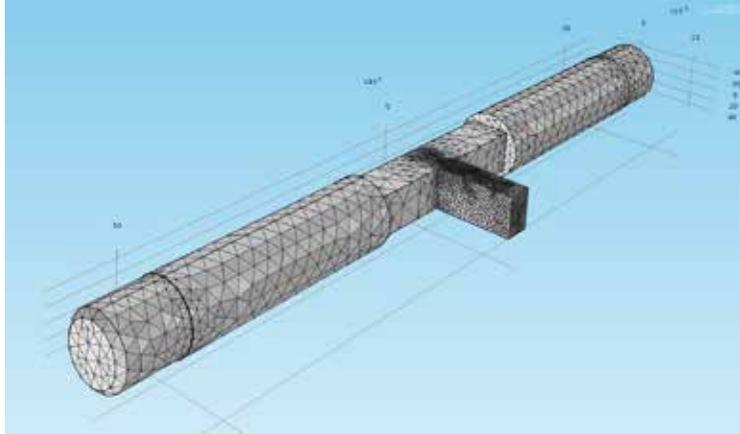


Fig. 3

Finite element mesh

The most popular modified Fourier equation, the so-called Cattaneo-Vernotte equation was used in this model to obtain the temperature field:

$$\rho c \frac{\partial T}{\partial t} + \rho c \tau \frac{\partial^2 T}{\partial t^2} = k \nabla^2 T + Q \quad (17)$$

The internal heat generation due to electromagnetic heat loss can be calculated by the following equation:

$$Q = \mathbf{J} \cdot \mathbf{E}, \quad (18)$$

where  $\mathbf{J}$  is the electric current density and  $\mathbf{E}$  is the electric field, which were calculated from the

Maxwell's equations.

The initial condition is assumed to be 20°C at all nodes in the mesh. The material properties for the AISI 321 structural steel used in this demonstration case are given in Tab. 2.

Tab. 2 Material parameters

No-tation	Parameter	Expression	Dimension
k	Thermal conductivity	$k(T) = 10.77298 + 0.01516699 \cdot T$	$\frac{W}{mK}$
C	Specific heat	$C(T) = 399.0295 + 0.3670225 \cdot T - (2.513748E-4) \cdot T^2 + (9.003721E-8) \cdot T^3$	$\frac{J}{kgK}$
$\rho$	Density	7850	$\frac{kg}{m^3}$
$\tau$	Relaxation time	$10^{-9}$	s
$\epsilon_r$	Relative permittivity	1.0008	-
$\sigma$	Electrical conductivity	$4 \cdot 10^{-6}$	$\frac{S}{m}$

The coupled electrical and heat transfer model is shown in Fig. 4.

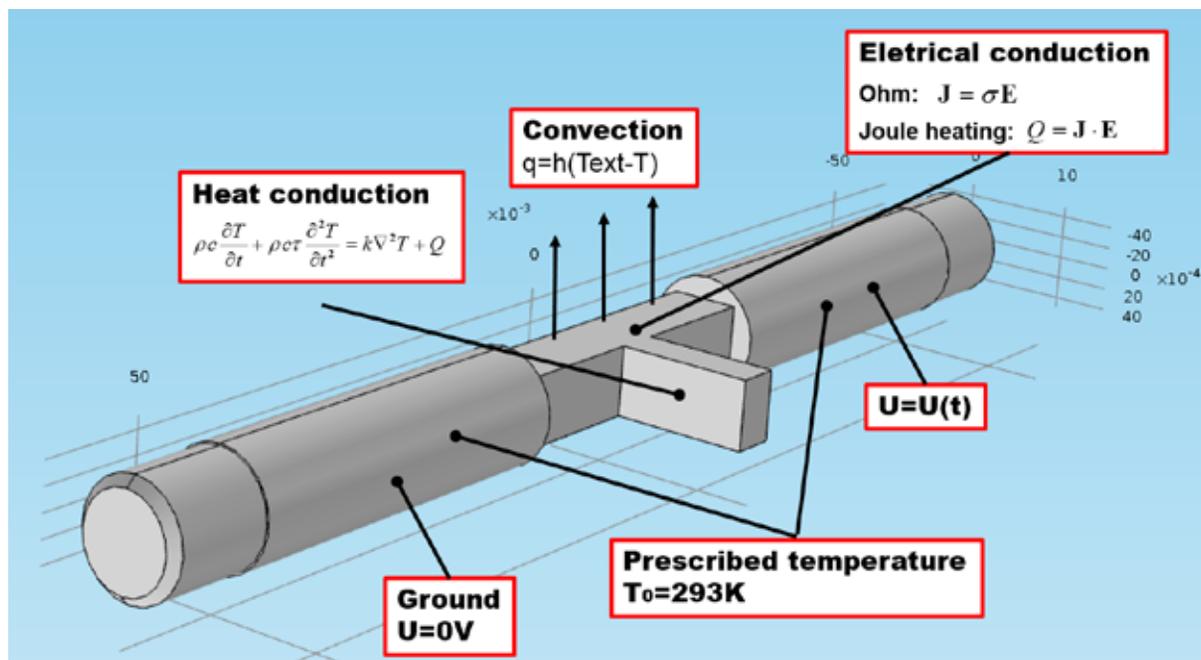


Fig. 4

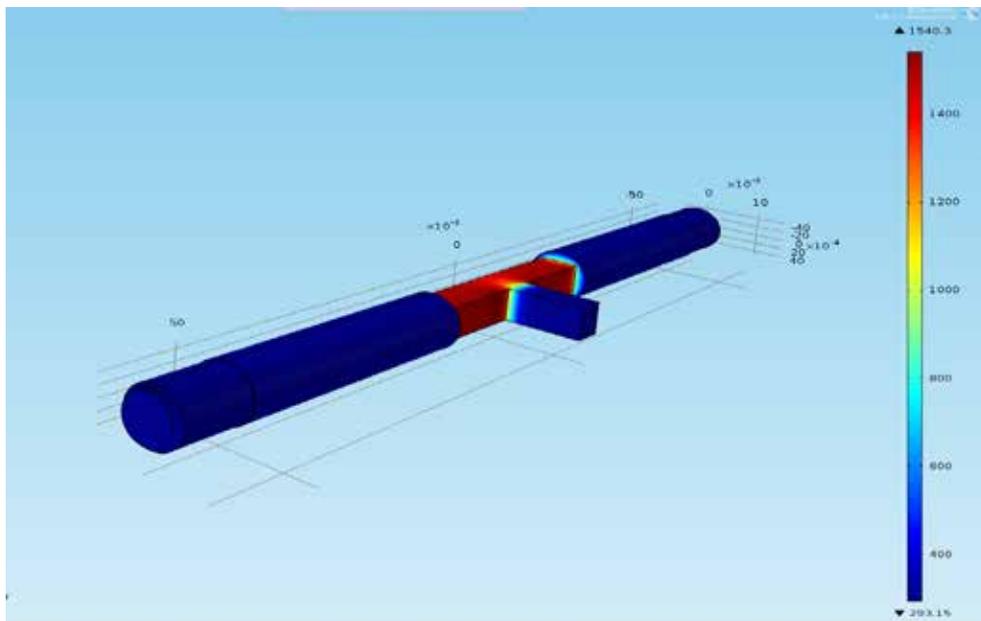
Governing equations and boundary conditions of the model

Tab. 3 Parameters of the heat transfer model

Notation	Parameter name	Parameter value	Dimension
$T_{init}$	Initial temperature	293	$K$
$T_{ext}$	External temperature	293	$K$
$h$	Heat transfer coefficient	2	$\frac{W}{m^2 K}$

## Results

Fig. 5 depicts the temperature field in the specimen at time  $t = 0,4$  s. In this demonstration calculation the thermal relaxation time were considered  $\tau = 10^{-9}$  s. Fig. 6 shows the temperature evolution in the perpendicular part of the specimen in selected point (T1-T5).



*Fig. 5 3-D plot of the temperature field  $t=0.4$  s, showing temperature growth due to electromagnetic heat source*

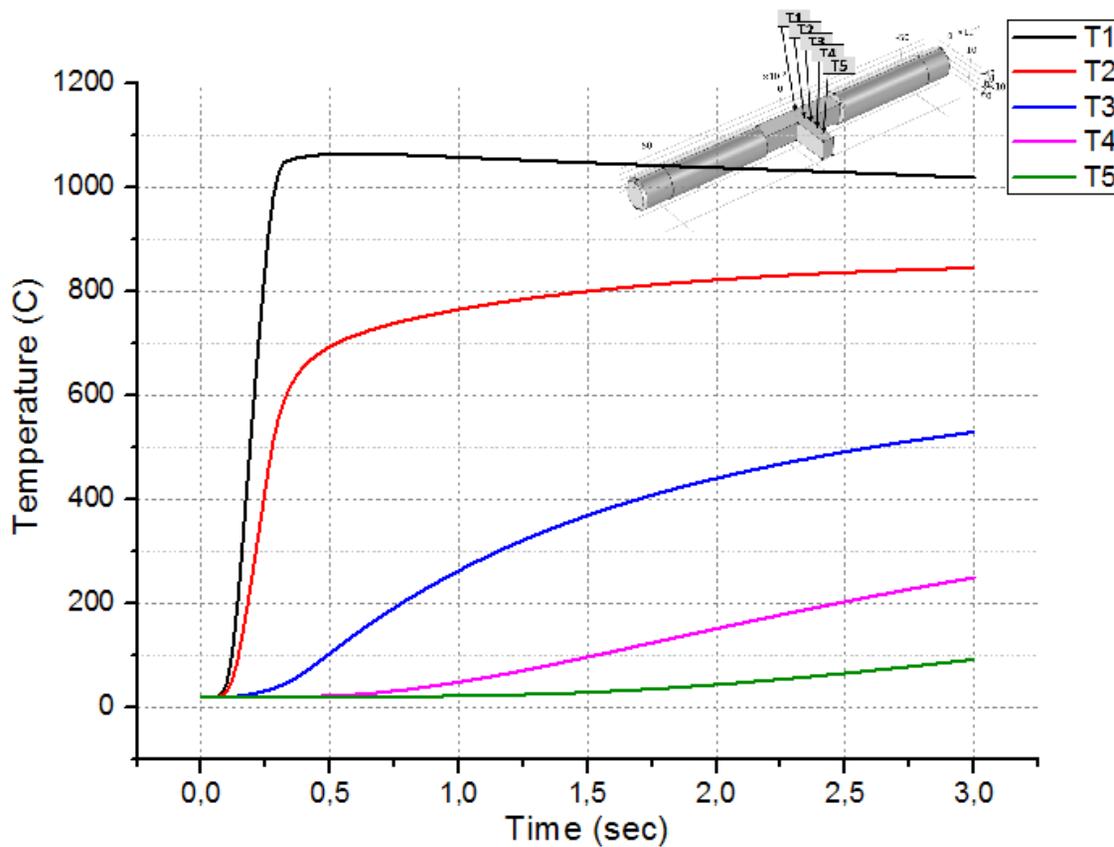


Fig. 6 Nodal temperature growth in the perpendicular part of the specimen due to heat conduction ( $\tau=10^{-9}$ )

## Conclusion

After summarizing some theoretical aspects on the modified equations of heat conduction finite element model was developed to simulate a thermal shock experiment of austenitic stainless steel. Though other models are available, the hyperbolic Cattaneo-Vernotte heat equation has been used in our computations, as there are many theories which validate it. In the coupled electrical-thermal model temperature dependent material properties were considered.

By testing the behavior of the developed finite element model we can concluded that, it is ready for the future work which will be directed toward the development of an optimization method, based on experimental data to obtain the parameters of the modified heat conduction equation.

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